

# Damping of Temperature Fluctuations Using Porous Matrices

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*This article examines the concept of utilizing the thermal attenuation characteristics of porous matrices and their thermal flywheel effect in damping the air temperature fluctuations for highly temperature-sensitive applications. The mathematical formulation of the problem in a dimensionless form is presented together with the relevant boundary conditions. The periodic temperature solution at a given matrix section has shown that the amplitude will be reduced by a logarithmic decrement and that the temperature cycle possesses a phase angle lag which depends on various flow and material properties, as well as the frequency of the temperature fluctuations. The effect of different material properties for porous matrix selection was examined by a numerical example.*

## I. Introduction

One of the major operation requirements for the frequency and timing subsystem for the Deep Space Network tracking antennas is the installation of a highly temperature-sensitive hydrogen maser oscillator to keep the microwave energy at a stable frequency. The maser oscillator is kept at a fixed predetermined temperature and is enclosed by several shields to isolate it from the effects of ambient temperature fluctuations. The hydrogen maser is housed in a special room which is air-conditioned by a conventional air-handler (fan-coil) unit. However, the system is thermally controlled to tolerate a very small periodic temperature fluctuation. This thermal protection was imposed in order to limit the operational frequency variations or drift to a minimum.

Although the early temperature control design of the air conditioner of the maser room was satisfactory, a better refinement of the temperature control has recently been sought to improve the system's operationability. Currently, several solutions are being investigated to reduce the frequency and temperature drifts due to the fluctuating room occupancy, maser wattage, ambient temperature, and solar irradiancy.

One of the suggested solutions is to let the conditioned air leaving the fan-coil unit pass through a porous matrix acting as a thermal flywheel which damps the temperature fluctuations up to the desired degree. This article addresses the mathematical analysis of the problem and the correlation between the relevant parameters. The objective is to obtain a

solution for the temperature distribution of an unsteady one-dimensional flow with constant transport properties through a porous matrix in which conduction and convection play a major role. Of major concern to the designer is the temperature profile at the fluid exit section, whereafter the fluid is directed to the maser oscillator. After the problem is analytically formulated, the system differential equation is solved and presented in Appendix A. The method of computations is given and illustrated by a numerical example. The matrix preliminary design procedure is also discussed briefly to acquaint the designer with the sequence to be followed.

## II. Analysis

Consider an element of a porous matrix as shown in Fig. 1 with a thickness  $dx$  placed at a distance  $x$  from the inlet fluid section, with properties  $C$ ,  $\rho$ , and  $K$  for the specific heat, density and thermal conductivity, respectively. The assumptions and the idealizations made in the analysis are as follows:

- (1) The heat conduction and fluid flow through the porous matrix are one-dimensional. The cross sectional area of matrix is denoted by  $A$ .
- (2) Thermal, physical, and transport properties of the matrix and the fluid are assumed to be spatially uniform over the entire control volume and independent of the operating temperature fluctuations.
- (3) The fluid temperature at the entrance section ( $x = 0$ ) is assumed to be sinusoidal. This is considered an adequate approximation to a temperature profile with a periodic drift due to the oscillations of ambient air temperatures, solar intensity, etc.
- (4) The matrix boundary is assumed to be well insulated such that no heat exchange is taking place along the matrix length with environment.

Denoting the assumed uniform porosity (ratio of fluid volume in pores to matrix volume) by  $P$ , the fluid properties by the subscript ( $f$ ), and the solid matrix properties by the subscript ( $s$ ), the conservation of energy for the unsteady one-dimensional flow can be written as follows, for the fluid flowing in the positive  $x$ -direction

$$P K_f \frac{\partial^2 T_f}{\partial x^2} - \frac{\dot{m}_f C_f}{A} \frac{\partial T_f}{\partial x} + H \zeta (T_s - T_f) = P C_f \rho_f \frac{\partial T_f}{\partial \tau} \quad (1a)$$

where  $\dot{m}_f$  is the fluid mass flow rate,  $H$  is the average heat transfer coefficient between the solid matrix and the fluid in the pores and  $\zeta$  is the average fluid surface area of pores per

unit matrix volume. The first term in the left hand side of Eq. (1a) represents the fluid conduction; the second term represents convection by the moving fluid; and the third term represents the heat gained due to the convection-radiation exchange between the fluid in the pores and matrix solid. Note that in formulating Eq. (1a), the effective fluid cross sectional area is  $PA$  and the effective elementary volume is  $PA dx$ , assuming that the pores are uniform and could be lumped together to form only one large pore next to a one lumped solid.

For the solid matrix, on the other hand, the energy equation is written as

$$(1 - P) K_s \frac{\partial^2 T_s}{\partial x^2} - H \zeta (T_s - T_f) = (1 - P) C_s \rho_s \frac{\partial T_s}{\partial \tau} \quad (1b)$$

The first term in the left hand side of Eq. (1b) represents the solid matrix conduction and the second term represents the heat lost to the fluid by convection-radiation exchange. Equations (1a) and (1b) represent a system of two simultaneous, partial differential equations in the two functions  $T_f(x, \tau)$  and  $T_s(x, \tau)$ .

A cursory look at Eqs. (1a) and (1b) shows that the solution of fluid and solid matrix temperatures will be almost identical if the fluid rate is very small and that the heat-convection-radiation term is small. Although Eq. (1a) and (1b) could have been solved for  $T_f$  and  $T_s$  in sufficient detail and accuracy, almost all analytical studies of transpiration-cooled matrices avoid the resulting complexity by making an additional simplifying assumption. The latter is to treat the temperatures  $T_f$  and  $T_s$  as equal at any position throughout the flow (Refs. 1-6). This assumption may not be very accurate for matrix heat exchangers (regenerators) with large fluid flow rates or with a high rate of change of fluid temperatures. However, this idealization in the mathematical model is still considered of utmost value as described in Refs. 1-6.

## III. Governing Differential Equation

By adopting the equal temperature assumption given in the last section, Eqs. (1a) and (1b) could be summed to form the new system differential equation in the temperature  $T(x, \tau)$  as

$$K_e \frac{\partial^2 T}{\partial x^2} - G_f C_f \frac{\partial T}{\partial x} = \rho_e C_e \frac{\partial T}{\partial \tau} \quad (2)$$

where the subscript (*e*) refers to the effective fluid-solid matrix properties,  $G_f$  is the fluid mass flux ( $\dot{m}_f/A$ ) and the effective properties  $K_e$  and  $\rho_e C_e$  are determined from:

$$\left. \begin{aligned} \rho_e C_e &= \rho_f C_f P + \rho_s C_s (1 - P) \\ K_e &= K_f P + K_s (1 - P) \end{aligned} \right\} \quad (3)$$

By dividing Eq. (2) by  $K_e$ , the differential equation will be reduced to

$$\frac{\partial^2 T}{\partial x^2} - \frac{G_f C_f}{K_e} \frac{\partial T}{\partial x} = \frac{1}{\alpha_e} \frac{\partial T}{\partial \tau}$$

where  $\alpha_e$  is the “effective” thermal diffusivity for the matrix, defined by

$$\alpha_e = K_e / \rho_e C_e \quad (4)$$

In order to obtain a general solution to this periodic heat-transfer problem regardless of the range of operating conditions or the physical units used in computations, a dimensionless form of Eq. (2) is sought. The dimensionless parameters for temperature  $\theta$ , time  $N$  and distance  $X$ , were finally selected, after making several dimensioning and substitution trials, to be as follows:

$$\left. \begin{aligned} \theta &= \frac{T(x, \tau) - T_m}{\Delta_0} \\ N &= f \tau \\ X &= x \sqrt{f / \alpha_e} \end{aligned} \right\} \quad (5)$$

where the amplitude  $\Delta_0$ , the mean temperature  $T_m$  and the frequency  $f$  are depicted in Fig. 2. Substituting in Eq. (2) using Eq. (5), the dimensionless form of the system's partial differential equation can be written for the temperature  $\theta(X, N)$  after some manipulation as

$$\frac{\partial^2 \theta}{\partial X^2} - F \frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial N} \quad (6)$$

where  $F$  is a dimensionless flow parameter, defined as

$$F = \frac{G_f C_f}{K_e} \sqrt{\frac{\alpha_e}{f}} \quad (7)$$

At zero flow factor  $F$ , which means at no flow condition or for the case of a solid surface, the differential equation is reduced to the one-dimensional transient heat transfer form with its known solution (Ref. 7). The existence of the term  $F(\partial\theta/\partial X)$ , however, makes the solution procedure somewhat different.

## IV. General Solution

The partial differential equation, Eq. (6), could be solved by using the separation of variables procedure where the dimensionless temperature  $\theta(X, N)$  is written in general as the product of the two functions  $\phi(X)$  and  $\psi(N)$ ; i.e.,

$$\theta(X, N) = \phi(X) \cdot \psi(N) \quad (8)$$

Substituting in Eq. (6), using the expression in Eq. (8) will yield, after dividing by  $\theta$  for both sides,

$$\frac{\phi''(X) - F \phi'(X)}{\phi(X)} = \frac{\psi'(N)}{\psi(N)} \quad (9)$$

The right hand side of Eq. (9) is a function of  $N$  only, and its left hand side is a function of  $X$  only, which means that both sides must be equal to a common constant. The latter can be any one of four possibilities: a zero, a real positive number ( $\lambda^2$ ), a real negative number ( $-\lambda^2$ ) or an imaginary number ( $\pm \lambda^2 i$ ). The first three possibilities are rejected since they will result in a nonoscillatory time solution at any position  $X$ , which does not fit the problem boundary conditions. The fourth possibility, ( $\pm \lambda^2 i$ ), is the only choice which requires the solution of the two linear differential equations

$$\left. \begin{aligned} \phi''(X) - F \phi'(X) - (\pm i \lambda^2) \phi(X) &= 0 \\ \psi'(N) - (\pm i \lambda^2) \psi(N) &= 0 \end{aligned} \right\} \quad (10)$$

At this stage, two solutions will be generated in solving Eq. (10), using in one the positive sign ( $+\lambda^2 i$ ) and using in the other the negative sign ( $-\lambda^2 i$ ) of the imaginary constant. This is explained in Appendix A in detail. The general solution of  $\theta(X, N)$  is expressed in Appendix A as Eq. (A-14), which is rewritten as

$$\theta = e^{aX} [B \cos(\lambda^2 N - bX) + D \sin(\lambda^2 N - bX)] \quad (11)$$

where  $B$ ,  $D$  are arbitrary constants, and  $a$ ,  $b$  are parameters given by Eq. (A-6).

## V. Boundary Conditions

At the fluid entrance section ( $X = 0$ ), the periodic temperature fluctuations, shown in Fig. 2 is written as:

$$T(0, \tau) = T_m + \Delta_0 \sin 2\pi f \tau$$

or in dimensionless form as

$$\theta(0, N) = \sin 2\pi N \quad (12)$$

Substituting in Eq. (11) at  $X=0$  using Eq. (12) and comparing the sine and cosine coefficients, then

$$\begin{aligned} B &= 0 \\ D &= 1 \\ \lambda^2 &= 2\pi \end{aligned} \quad (13)$$

Hence, the solution for the dimensionless temperature ( $\theta$ ) becomes

$$\theta = \exp(aX) \sin(2\pi N - bX) \quad (14)$$

This means that as  $X$  increases (away from the entrance section), the amplitude of the sinusoidal fluctuations will decrease ( $a$  is always a negative quantity). Note that both the fluid and the matrix are assumed to be approximately at the same temperature.

For the periodic temperature fluctuations, at any section  $X$ , the temperature  $T$  should be such that

$$T(x, \tau) = T(x, \tau + n/f)$$

or in dimensionless form

$$\theta(X, N) = \theta(X, N + n) \quad (15)$$

where  $n$  is any integer representing the number of cycles. One can see that this boundary condition is satisfied when Eq. (14) is used for any number of cycles  $n$ .

A simple corollary could be derived from Eq. (14) about the mean temperature of the matrix at any distance  $x$ . By obtaining the mean value of the temperature  $T(x, \tau)$  during one cycle, i.e.,  $[f \int_0^{1/f} T(x, \tau) d\tau]$ , the mean temperature at the entrance section,  $T_m$  will be the same for any other

section. The amplitude, however, will decrease as mentioned above.

## VI. Temperature Profile

The solution of the temperature profile along a one-dimensional flow through a porous matrix is represented by Eq. (14) and is sketched in Fig. 3. The temperature cycle at a distance  $x$  from the fluid inlet section will experience a time lag  $\tau^*$ , determined from Eqs. (5) and (14) as

$$\tau^* = \frac{bx}{2\tau \sqrt{f\alpha_e}} \quad (16)$$

The amplitude, on the other hand, will monotonically decrease by an exponential function, as  $x$  increases. At a distance  $L$  from the fluid inlet section, the temperature amplitude will be  $\Delta_L$ , given by the logarithmic decrement written as

$$\ln \frac{\Delta_L}{\Delta_0} = aL \sqrt{\frac{f}{\alpha_e}} \quad (17)$$

where the parameters  $a$  and  $b$  are determined from Eq. (A-6) as

$$\left. \begin{aligned} a &= \frac{F}{2} - \frac{1}{2} (F^4 + 64\pi^4)^{1/4} \cos \delta/2 \\ b &= \frac{1}{2} (F^4 + 64\pi^4)^{1/4} \sin \delta/2 \end{aligned} \right\} \quad (18)$$

and

$$\delta = \tan^{-1} \frac{8\pi}{F^2}$$

The relationship between the parameter  $a$  and the flow factor  $F$  can be directly obtained from Eq. (18) as

$$a = \frac{F}{2} - \frac{1}{2\sqrt{2}} \sqrt{F^2 + \sqrt{(F^4 + 64\pi^2)}} \quad (19)$$

The relationship between  $a$  and  $F$  is plotted as shown in Fig. 4. The parameter  $a$  is always negative. Increasing the flow factor  $F$  will decrease the absolute value of  $a$ , thus requiring a larger matrix length  $L$  to damp a given amplitude ratio. Also, as seen from Eqs. (7) and (17), the use of a porous matrix having large

thermal conductivity ( $K_e$ ) is preferred in order to reduce the matrix length  $L$ .

## VII. Design Sequence

To illustrate the use of the above equations and the sequence followed, the following numerical example is given. Suppose that the air temperature profile at the matrix entrance section ( $X=0$ ) has an amplitude  $\Delta_0$  of  $0.05^\circ\text{C}$  and that the air temperature frequency  $f$  could be approximated as four cycles/hr; then in order to reduce the air temperature amplitude down to  $0.025^\circ\text{C}$  at the exit section, a porous matrix of length  $L$  will be determined according to the following steps:

- (1) Select the material to be used for the solid matrix and determine the transport properties  $\rho$ ,  $C$ ,  $K$ , and  $\alpha$  at the mean operating temperature for both the fluid and solid matrix.
- (2) Determine the porosity  $P$  either from specifications or by choosing a trial value for the design to be changed later. The equivalent properties  $\rho_e$ ,  $C_e$ ,  $K_e$ , and  $\alpha_e$  could be determined using Eqs. (3) and (4).
- (3) From the temperature frequency  $f$ , mass flux  $G_f$  and properties  $C_f$ ,  $K_e$  and  $\alpha_e$ , determine the dimensionless flow factor  $F$  using Eq. (7).
- (4) Knowing the flow factor  $F$  the parameter  $a$  is evaluated from Eq. (19).
- (5) Determine the length  $L$  required to satisfy the logarithmic decrement  $[\ln(\Delta_L/\Delta_0)]$  of Eq. (17).

- (6) Repeat the above steps for different porosity values, matrix materials, air mass flow, frequency, etc., for parameterization.

Table 1 lists the results of one design trial at an arbitrary porosity of 0.2. The air mass flux  $G_f$  is calculated as  $10,000 \text{ kg}/(\text{hr} \cdot \text{m}^2)$  at an air velocity of about  $454 \text{ ft}/\text{min}$ , and at an air temperature of  $20^\circ\text{C}$  ( $68^\circ\text{F}$ ). Four types of matrix materials were tried in Table 1, cotton wool, steel, brass and aluminum.

It can be seen from Table 1 that the effective diffusivity  $\alpha_e$  is nearly independent of the porosity  $p$  for metallic matrices due to the large solid properties  $K_s$  and  $\rho_s C_s$  compared to the fluid (air). However, as the porosity decreases, the equivalent thermal conductivity  $K_e$  decreases and the flow factor  $F$  decreases, which in turn will lead to a smaller matrix length  $L$  for damping the temperature fluctuations. Also, the smaller the mass flux  $G_f$ , and the higher the frequency  $f$ , the smaller the matrix length  $L$  as evidenced from Eqs. (7), (17), and (19). For the above numerical example, a brass matrix with a porosity of 0.2 and a length of  $0.62 \text{ m}$  would be theoretically sufficient to damp the temperature amplitude to one-half.

In practice, a somewhat longer matrix would be required to offset the differences between matrix and pores temperatures, which are neglected in Section II. The order of magnitude of the errors caused by using the approximate differential equation, Eq. (2), instead of the accurate one, Eq. (1), will be addressed in another report.

**Table 1. Effect of matrix material on required matrix size for the given example**

Matrix material	Solid density, $\text{kg}/\text{m}^3$	Solid sp. heat $C_s$ , $\text{Wh}/\text{kg}^\circ\text{C}$	Solid thermal cond. $K_s$ , $\text{W}/\text{m}^\circ\text{C}$	Equivalent solid thermal diffusivity $\alpha_s$ , $\text{m}^2/\text{hr}$	Equivalent <sup>a</sup> thermal cond. $K_e$ , $\text{W}/\text{m}^\circ\text{C}$	$\alpha_e$ , $\text{m}^2/\text{hr}$	Flow factor $F$	$a$	$L$ , $\text{m}$
Cotton wool	80	0.360	0.042	0.0015	0.039	0.0017	1486	$-3 \times 10^{-7}$	47,600
Steel	7750	0.134	39.5	0.0381	31.6	0.0382	8.6	-0.06	1.13
Brass	8500	0.107	106.4	0.1170	85.1	0.117	5.6	-0.19	0.62
Aluminum	2700	0.249	196.5	0.2925	157.2	0.2925	4.8	-0.27	0.69

<sup>a</sup>Properties of air at  $20^\circ\text{C}$ :

$$\rho_f = 1.205 \text{ kg}/\text{m}^3, K_f = 0.0259 \text{ W}/\text{m}^\circ\text{C}$$

$$\alpha_f = 0.0771 \text{ m}^2/\text{hr}, C_f = 0.279 \text{ Wh}/\text{kg}^\circ\text{C}$$

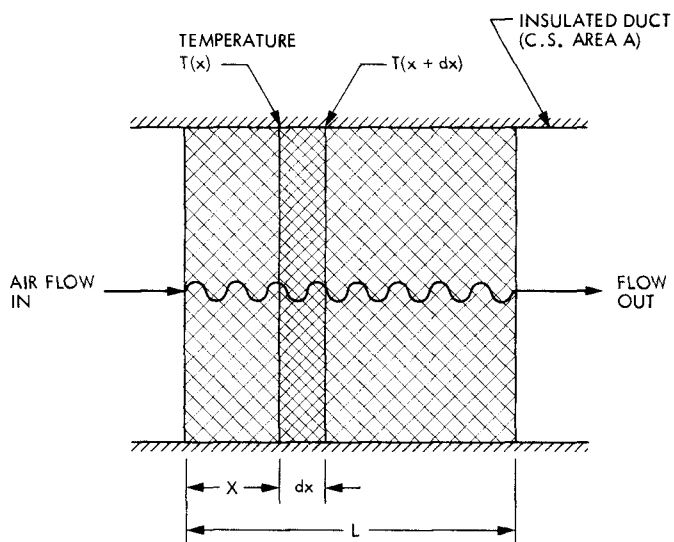


Fig. 1. Fluid through a porous matrix

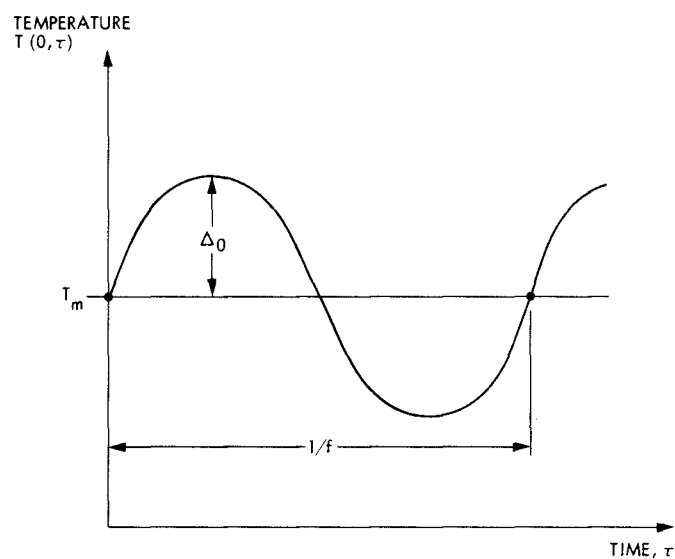


Fig. 2. Temperature fluctuations at the fluid entrance section ( $X=0$ )

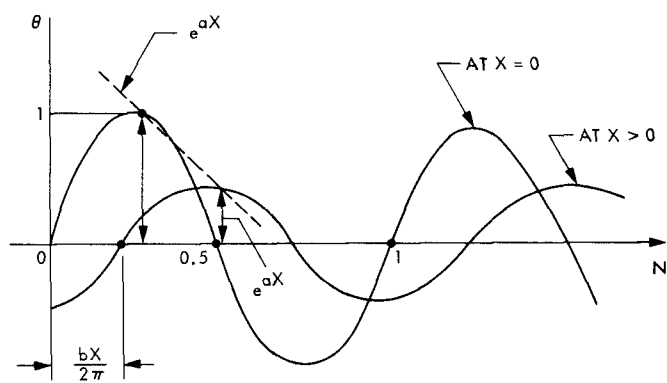


Fig. 3. Temperature oscillation at a distance  $X$  from the fluid entrance section

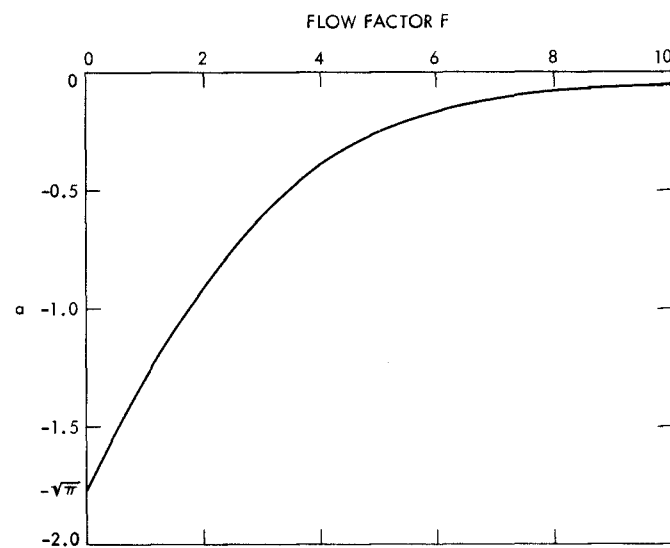


Fig. 4. Relationship between  $a$  and flow factor  $F$  for small values of  $F$

## Appendix A

### Solution of the System Differential Equation

The two functions  $\phi(X)$  and  $\psi(N)$ , expressed in Eq. (10), could be obtained by using either the positive imaginary constant ( $i\lambda^2$ ) or its negative conjugate ( $-i\lambda^2$ ). The two possible solutions can be generated for each case as explained next.

#### I. Taking the Positive Constant (+ $i\lambda^2$ )

In this case, Eq. (10), in the text, can be written as

$$\left. \begin{aligned} \phi''(X) - F\phi'(X) - i\lambda^2\phi(X) &= 0 \\ \psi'(N) - i\lambda^2\psi(N) &= 0 \end{aligned} \right\} \quad (\text{A-1})$$

The two functions  $\phi(X)$  and  $\psi(N)$  will then be solved as

$$\left. \begin{aligned} \phi(X) &= B_1 \exp(S_1 X) + B_2 \exp(S_2 X) \\ \psi(N) &= B_3 \exp(i\lambda^2 N) \end{aligned} \right\} \quad (\text{A-2})$$

where  $B_1$ ,  $B_2$ , and  $B_3$  are arbitrary constants and  $S_1$ ,  $S_2$  are the roots of the auxiliary equation

$$S^2 - FS - i\lambda^2 = 0 \quad (\text{A-3})$$

The roots  $S_1$  and  $S_2$  can be determined by solving Eq. (A-3) as

$$S_{1,2} = \frac{F \pm \sqrt{F^2 + 4i\lambda^2}}{2} \quad (\text{A-4})$$

Using the complex number algebra, the square root in Eq. (A-4) can be rewritten in terms of the angle parameter  $\delta$ . Accordingly, the roots  $S_1$  and  $S_2$  are expressed as:

$$\left. \begin{aligned} S_1 &= r + bi \\ S_2 &= a - bi \end{aligned} \right\} \quad (\text{A-5})$$

where

$$\left. \begin{aligned} r &= \frac{F}{2} + \frac{1}{2} (F^4 + 16\lambda^4)^{1/4} \cos \delta/2 \\ a &= \frac{F}{2} - \frac{1}{2} (F^4 + 16\lambda^4)^{1/4} \cos \delta/2 \\ b &= \frac{1}{2} (F^4 + 16\lambda^4)^{1/4} \sin \delta/2 \end{aligned} \right\} \quad (\text{A-6})$$

$$\delta = \tan^{-1} \frac{4\lambda^2}{F^2}$$

and

The solution of the dimensionless temperature  $\theta_+$  using the positive sign can thus be written as

$$\theta_+ = \exp(i\lambda^2 N) [B'_1 \exp(r + bi)X + B'_2 \exp(a - bi)X] \quad (\text{A-7})$$

where the arbitrary constant  $B_3$  is combined with the constants  $B_1$  and  $B_2$

#### II. Taking the Negative Constant ( $-i\lambda^2$ )

In this case, Eq. (10) in the text can be written as:

$$\left. \begin{aligned} \phi''(X) - F\phi'(X) + i\lambda^2\phi(X) &= 0 \\ \psi'(N) + i\lambda^2\psi(N) &= 0 \end{aligned} \right\} \quad (\text{A-8})$$

The functions  $\phi(X)$  and  $\psi(N)$  after solving Eq. (A-8) will be written as

$$\left. \begin{aligned} \phi(X) &= B_4 \exp(S'_1 X) + B_5 \exp(S'_2 X) \\ \psi(N) &= B_6 \exp(-i\lambda^2 N) \end{aligned} \right\} \quad (\text{A-9})$$

where  $B_4$ ,  $B_5$ , and  $B_6$  are arbitrary constants and  $S'_1$ ,  $S'_2$  are the roots of the auxiliary equation

$$S^2 - FS + i\lambda^2 = 0 \quad (\text{A-10})$$

The roots  $S'_1$  and  $S'_2$  can be determined by a procedure similar to that used for  $S_1$  and  $S_2$ . Therefore,

$$\left. \begin{aligned} S'_1 &= r - bi \\ S'_2 &= a + bi \end{aligned} \right\} \quad (\text{A-11})$$

where the parameters  $a$ ,  $b$  and  $r$  are as defined by Eq. (A-6). The solution of the dimensionless temperature  $\theta_-$  using the negative sign can be reduced to

$$\theta_- = \exp(-i\lambda^2 N) [B'_4 \exp(r - bi)X + B'_5 \exp(a + bi)X] \quad (\text{A-12})$$

where the arbitrary constants  $B'_4$  and  $B'_5$  combine the constant  $B_6$  with  $B_4$  and  $B_5$ .

### III. General Solution

By adding the two possible solutions in Eqs. (A-7) and (A-12), the general solution of the dimensionless temperature ( $\theta$ ) can be reduced, after some manipulation, to

$$\begin{aligned} \theta &= \exp(rX) \cdot \{B'_1 \exp[(\lambda^2 N + bX)i] \\ &+ B'_4 \exp[-(\lambda^2 N + bX)i]\} \\ &+ \exp(aX) \{B'_2 \exp[(\lambda^2 N - bX)i] \\ &+ B'_5 \exp[-(\lambda^2 N - bX)i]\} \end{aligned} \quad (\text{A-13})$$

Since the value of  $r$  (from Eq. (A-6)), is always positive, no solution would exist from the first term in Eq. (A-13) where it is not conceivable to have an infinite temperature  $\theta$  at  $X$  equals  $\infty$ . This will lead to the elimination of the arbitrary constants  $B'_1$  and  $B'_4$ . The same conclusion would have been reached for the second term in Eq. (A-13) if the value of  $a$  was positive at any time. It can be easily proven that the value of  $a$  from Eq. (A-6) is always negative at any value of  $F$  or  $\lambda$ . The general solution of  $\theta$ , therefore, will be only the second term of Eq. (A-13), which can be rewritten in the trigonometric form as

$$\theta = \exp(aX) [B \cos(\lambda^2 N - bX) + D \sin(\lambda^2 N - bX)] \quad (\text{A-14})$$

where  $B$  and  $D$  are arbitrary constants to be determined further from the initial and boundary conditions.



## Definition of Symbols

$A$	cross section area	$N$	number of cycles
$B$	arbitrary constant	$p$	porosity (pore volume/total volume)
$b$	parameter	$T$	temperature
$C$	specific heat	$X$	dimensionless distance
$D$	arbitrary constant	$x$	distance measured from the fluid entrance
$F$	dimensionless flow number	$\alpha$	thermal diffusivity = $K/\rho c$
$f$	frequency	$\Delta$	amplitude
$G$	mass flux (flow rate per unit area)	$\delta$	angle parameter
$H$	average heat transfer coefficient between solid matrix and fluid in pores	$\zeta$	average pore surface area per unit matrix volume
$i$	$\sqrt{-1}$	$\theta$	dimensionless temperature
$K$	thermal conductivity	$\lambda$	parameter
$L$	matrix thickness	$\rho$	density
$m$	mass flow rate	$\tau$	time elapsed
		$\phi, \psi$	functions

## Suffixes

$e$	“effective” property for matrix	$s$	solid matrix
$f$	fluid	$m$	mean or average

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